

Is there a Spiritual Significance to the number Phi (ϕ)?

By George Gantz^{1,2}

Introduction

We know that numbers are important in the natural world and particularly in understanding the natural world through science. From the Writings of Emanuel Swedenborg we learn that numbers are important in a spiritual sense as well. From beginning to end, the Word (Bible) is filled with numbers, and one of its books is so named. Swedenborg explains the significance of these numbers in many of his works. In the Apocalypse Explained (n. 336) he says that numbers or measures denote the quantity of a thing in the natural sense, and the quality of a thing in the spiritual sense. He also refers to the fact that numbers are used as a kind of language in the spiritual world, with each idea represented by a unique number, in a way that only those in the same heaven can understand. (Heaven and Hell, n. 263; Arcana Coelesta, n. 4495)

With this in mind, it may be interesting to explore the properties of a rather unique and special number, known as phi, or ϕ . This number has been pondered over the centuries, but continues to retain a remarkable mystery, even more of a mystery than its more famous relative “Pi” (π = the ratio of the circumference of a circle to its diameter). What conclusion can we draw about the spiritual significance, if any, of the number ϕ , from the backdrop of Swedenborg’s teachings?

About Phi - Φ

The number ϕ (pronounced either as “fie” or “fee”) is sometimes referred to as “the golden ratio”. Although it apparently is never referenced in the Word, it was known by the early Greeks (Livio, p. 24ff) and many have claimed it was used by the Egyptians at the time of the pyramids (Livio, p. 42ff.). In spite of its antiquity, ϕ is quite a

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² **Acknowledgements:** As inspiration for this paper I would like to acknowledge both a children’s talk by Bishop Louis King to the Boston Society on December 12, 2004, on the meaning of the number “two”, and the book by Mario Livio entitled The Golden Ratio, published by Broadway Books in 2002. Livio’s treatment of the topic is thorough and enjoyable, although he downplays the notion of a spiritual significance for the golden ratio (something which many great minds have espoused for centuries). Mathematical diagrams and formulas in this article are courtesy of Eric W. Weisstein, “Golden Ratio” from MathWorld - A Wolfram Web Resource. (<http://mathworld.wolfram.com/GoldenRation.html>)

remarkable number, and its properties continue to be explored and uncovered even in the latest discoveries of mathematics and science. (Livio, p. 114f, p. 206ff)

There are several ways to approach ϕ and its remarkable properties, but one of the simplest is with a calculator. Take your calculator and type in the approximate value of ϕ , 1.61803399. Square it, and then subtract the number one. Look familiar? The answer is ϕ . Now find its reciprocal by dividing into one (one divided by ϕ) – you get 0.61803399. Add one, and you get – once again – ϕ . Of all the infinite numbers in the universe, ϕ is the only number with these rather curious properties.

This calculator trick may seem cute, but the mathematics gets serious if you look at something more daunting – a repeating square root using only the number one, i.e. the square root of (one plus the square root of (one plus the square root of (one plus ...))). Take this expression to infinity and solve it - you get ϕ .

$$\phi = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}$$

Similarly, if you analyze the continued fraction of the number one, i.e. one plus (one divided by one plus (one divided by one plus (...))), taken to infinity, you also get ϕ .

$$\phi = [1, 1, 1, \dots] = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}$$

This unusual behavior continues when you examine an extremely important but simple numerical series known as the Fibonacci sequence, named for the Italian mathematician Leonardo Fibonacci who discovered it. This series starts with the number one, then adds the prior number to get the next one. Thus the series goes like this:

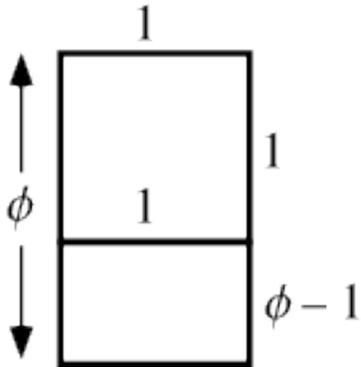
1; then 1+0=1; then 1+1=2; then 2+1=3; then 3+2=5; then 5+3=8 ... or:

1 – 1 – 2 – 3 – 5 – 8 – 13 – 21 – 34 – 55 – 89 – 144 – 233 – 377 ...

If you take the last number and divide it by the preceding one, you get an approximation of the number ϕ . If you repeat the series to infinity, the ratio of the two consecutive numbers in the sequence converges to equal ϕ .

The most interesting observation about these three seemingly unrelated calculations is that they involve operations on the number one. Through the infinite repetition of addition, division, or square roots, using just the number one, we arrive at the same number – ϕ .

These examples may seem, to the non-mathematical, to be interesting, but largely irrelevant. But consider the figure in art and aesthetics known as “the golden rectangle”.

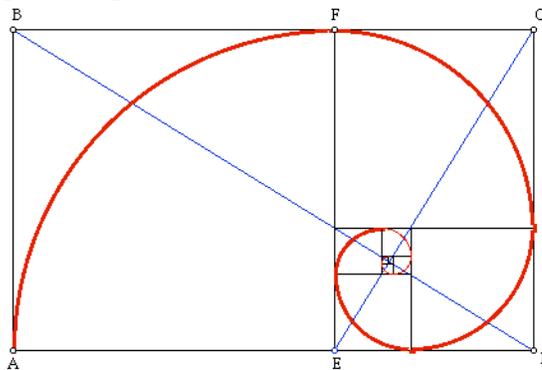


A golden rectangle is one where the long side is ϕ times bigger than the short side. If you then take the length of the short side and bisect the long side, you create two shapes inside the rectangle – a square, and another golden rectangle. The short side of the smaller rectangle is now $(1/\phi)$ times the long side. As we know from the calculator exercise, this is also equal to $(\phi-1)$.

Take the smaller rectangle and you can do the same thing to create another, even smaller golden rectangle inside. You can repeat this process to infinity, or you can go the other way to make a bigger and bigger rectangle. The ratio of the longer side to the shorter side of a golden rectangle is always equal to ϕ .

The golden rectangle is used frequently in art and architecture because, in some way, the proportions are aesthetically pleasing. This “golden” proportion, and its geometrical cousins (which include the regular pentagon and various other shapes) are found in the pyramids of Egypt, in the Parthenon, and in many works of art. Leonardo da Vinci appears to have been strongly influenced by the contemporary mathematics concerning the “divine proportion”. (Livio, p131ff)

However, the divine proportion is even more prevalent in nature than it is in art. Perhaps the most frequently cited occurrence is the beautiful nautilus shell, where each succeeding chamber is ϕ times larger than the preceding one, with the resulting shape drawing the arc of a perfect spiral.



Natural reproductive processes, such as the breeding of rabbits, follow the Fibonacci sequence, where ultimately each number in the series is ϕ times bigger than the last. This unique relationship appears in the overlapping petals in a rose, the arrangement and growth of leaves, the patterns and growth of crystals, the patterns on a pineapple or in the seeds on a sunflower, the structure of the galaxies and the interactions of subatomic particles - all involve the golden ratio of ϕ . This remarkable number is ubiquitous in nature, and perhaps fundamental to all things that grow.

These natural phenomena continue to be studied in the various sciences today, but in the last decade a general understanding of the reason for ϕ 's prevalence in nature has emerged. (Livio p. 114f) As natural systems grow, they must use energy, and they will naturally tend to use the least amount of energy as they move from one state to the next. The minimization of the use of energy, or the achievement of maximum efficiency, is precisely determined in the mathematical qualities of the number ϕ . It could be described as the efficiency coefficient of the universe.

The Spiritual Significance of Numbers

Swedenborg, in discussing the use of words and numbers in heaven, says in Heaven and Hell, "All numbers do in fact correspond and have meaning depending on their correspondence, just as words do, but with the difference that numbers represent general entities and words specific ones. Since one general entity involves countless specific ones, numeric writing enfolds more mysteries than alphabetic writing." (HH, n. 263)

Elsewhere, in referring to the meaning of numbers in the Word, he says, in Arcana Coelesta, "numbers and measures signify things celestial and spiritual..... by the numbers and measures are signified holy things ..." (AC, n. 648) And again, "the signification of number as denoting quantity and quality, quantity in the natural sense and quality in the spiritual sense" (Apocalypse Explained, n. 336).

Given that numbers signify general spiritual things, holy things, and the quality of things, is there anything we can say about the spiritual significance of the number ϕ ? Clearly it's an important number in the natural world, and perhaps it signifies an important spiritual idea. But how do we determine that significance?

One clear feature of ϕ is that it is related in important ways to the number one. Take the infinitely repeating square root or infinite continued fraction of one, or add one infinitely to itself in the Fibonacci sequence, and you derive $-\phi$. Thus ϕ could be described as an infinite unfolding, whether through roots, fractions or addition, of the number one. So some of the significance of ϕ must be derived from the correspondence with the number one.

This approach is consistent with Swedenborg's analysis, since operations on numbers apparently do not modify the spiritual meaning, but accentuate it. "... but the half implies the same as the whole, for multiplication and division, where a like thing is involved, do not vary the thing itself as to what is essential." (AC, n. 3239) "In general it is to be known that numbers multiplied involve nearly the same thing as simple numbers, but what is more complete; and that numbers divided involve the like, but what is not so complete." (AC, n. 5291)

So we should start by examining the spiritual significance of the number one. This seems very simple. "God is One, and the Lord is that God." (Doctrine of Life, Chapter IX Title). "There is One God-Man From Whom All Things Are" (Divine Love and Wisdom, n. 23) "I am the Lord your God... you shall have no other Gods before

me.” (Exodus 20:1) The number one, in its purest sense, would seem to correspond with the Lord God himself, creator of heaven and earth.

But God is also infinite: “It is well known that God is infinite for He is called ‘the Infinite’; but He is called the ‘Infinite’ because he is Infinite. He is the infinite not from this alone that He is Esse and Existere in Himself, but because there are infinite things in him.” (DLW, n. 17)

Swedenborg also touches on another aspect of the number one, that being the concept of conjunction where two or more things make one whole. “Every unit is formed by the harmony of many components, and that such as is the harmony, such is the one, and that it is impossible for anything to subsist that is absolutely a one, but only a one that results from a harmony of component parts...” (AC, n. 457)

And further, “Esse and Existere in God-Man are One Distinctly ... because the one is possible along with the other one and not without..., it follows that they are one, but one distinctly... like soul and body. There can be no soul without its body, nor a body without its soul.... There is a uniting into one, hence it is that the one is the other mutually and reciprocally, and the one is that all-in-all of the other as in itself.” (Excerpted from DLW, n. 14 and 15)

What Can We Conclude

The passages above confirm the powerful conception of God as the infinite “ONE”, the conjunction of the infinite powers and properties of the divine into one perfect harmony. And from our mathematical analysis, we know that ϕ is the number that reflects an infinite unfolding of the number one - ϕ is the result of an infinite series of operations on the number one.

This suggests that ϕ may correspond spiritually to the Lord’s act of creation – to the Lord’s infinite love and wisdom, acting in perfect harmony, to unfold the finite universe. This spiritual creation proceeds from God the most infinite, to the most finite, the natural world. The derivation of the number ϕ is quite similar – from the most finite of numbers, the number “1”, which corresponds to the Lord, an infinite series of operations proceeds to its derivation – the number ϕ . Thus, ϕ is created through the infinite unfolding of the number one.

But ϕ , as the efficiency coefficient of the natural world, also corresponds spiritually to the qualities of harmony and efficiency, qualities which are manifest in the Lord’s perfect creation, and which are apparent in the natural beauty of this world.

Moreover, in the natural world, ϕ is most evident in the processes of growth and life, the most compelling natural expression of the Lord’s creation.

One other useful mathematical concept related to ϕ is what is called the perfect, or logarithmic, spiral. (Livio, p. 115ff) This is the spiral seen in the nautilus shell, in the head of a sunflower, in the trajectory of a falcon speeding towards its prey or in the arms

of a spiral nebula. Its special property is that its shape, no matter what section of the spiral you examine and no matter how detailed the examination, is always the same.

In a spiritual sense, this perfect spiral offers a beautiful image of regeneration – spiritual growth towards a perfect reciprocal union with God. The perfect spiral so apparent in the nautilus shell is an image of the perfect arc of regeneration, the process each one of us can move through in our journey to achieve conjunction with the Lord.

Epilogue

These observations are likely just the beginning of what could be a long and fruitful exploration of mathematical qualities and principles, in the light of Swedenborg's spiritual teachings. That there is a powerful connection between the physical world and spiritual truth is undeniable to anyone with faith in the Lord and his creation. And although the realms of science and spirituality often seem so distant, such an exploration could be useful in trying to bridge this divide.

Clearly, the analysis above can change one's perception of the physical processes and mathematical principles of the natural world. The mathematical puzzles of the number ϕ and its many manifestations in the world, such as the multiple spirals in the head of a sunflower, become an obvious and tangible expression of the Lord in his creation.

The exploration may also be helpful in working towards the state of innocence and a more direct understanding of the Lord which Swedenborg attributed to "the most ancient people."

"... every number has a meaning... I was told that they came forth from angelic speech, and that they sometimes expressed things by numbers... This was known to the most ancient people who were celestial men and conversed with angels, and hence they formed an ecclesiastical reasoning by means of numbers, which expressed universally the things expressed particularly in words. But what each number expressed was forgotten by their posterity, except what was signified by the simple numbers..." (AC, n. 5265)